Heat exchanger network synthesis with multiple utilities using a generalized stagewise superstructure with cross flows

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Abstract
Most of the existing simultaneous approaches for heat exchanger network synthesis assume that only one type of hot/cold utility is available to adjust the final temperatures. In this work, we propose a simultaneous mixed-integer nonlinear programming formulation based on a more generalized stagewise superstructure. We allow utilities to adjust the temperature of process streams in each stage. We also focus on including the certain that the previous stagewise superstructure and hyperstructure neglects. Using several examples, we demonstrate the significant advantages of our approach compared to those from the literature.

Keywords: heat exchanger network synthesis (HENS); multiple utilities; superstructure; mixed-integer nonlinear programming (MINLP); optimization.

1. Introduction
Without access to affordable renewable energy, the question of how best to utilize the limited fossil energy efficiently will remain a key question for the near future. The chemical process industry consumes extensive energy to produce utilities for heating and cooling process streams. Heat exchanger network synthesis (HENS) to match the supply and demand of heat in a chemical plant has gained intense and significant attention in both practice and research for several decades.

The basic HENS problem is to develop a heat exchanger network (HEN) with the minimum annualized investment and annual operating cost, which integrates given sets of hot and cold process streams with known initial and final temperatures, heat capacities, and flow rates, while the available utilities and their temperatures and cost coefficients are also given. The early models and approaches for HENS were sequential. In general, the sequential approach involves decomposing the HENS problem into several subproblems, which is typically accompanied by dividing the temperature range of the problem into temperature intervals. On the other hand, the simultaneous approach uses mathematical programming with total annualized cost (TAC) as its single and fundamental objective and holistically trade-offs various factors (utility usage, number and heat exchange areas) affecting TAC in a single step. Given the advances in optimization solvers and computational power, we consider it attractive to rely on this approach in this work.
Most of the current simultaneous approaches for HENS employ a stagewise superstructure (Yee and Grossmann, 1990) and assume that only one type of hot/cold utility is available. Also, even for the existing work dealing with multiple utilities (Isafiade and Fraser, 2008; Hasan et al., 2009), utilities are only allowed to adjust the final/end temperatures. However, multiple hot and cold utilities are common in real chemical processes. For instance, the use of steam at different levels (high, medium and low pressure), hot oil, hot water, different refrigerants, cooling water, and air abounds in practice. Then, the TAC of a HEN depends on not only the amounts and types of utilities, but also their placement, since not all hot (cold) utilities are hotter (colder) than the hottest (coldest) process streams. In fact, multiple utilities can be accommodated at every stage in the stagewise superstructure by treating them as process streams with variable flows. However, this increases model size. Ponce-Ortega et al. (2010) have developed an MINLP model based on the stagewise superstructure to address the optimal location of multiple utilities. However, the inherent limitations of the stagewise superstructure may still lead to inferior solutions.

To overcome those limitations of stagewise superstructure, Floudas and Ciric (1989) developed a hyperstructure, which is similar to single-stage stream superstructures of Floudas et al. (1986), allowing non-isothermal mixing and cross flow. Subsequently, Ciric and Floudas (1991) combined this hyperstructure with the pinch-based transshipment model of Papoulias and Grossmann (1983) to simultaneously select optimal HEN configuration. However, the hyperstructure model does not allow cyclic matching, which may help to decrease HE area at the expense of more HE units in some cases and give superior HENs with less TAC. Recently, Huang and Karimi (2012) proposed a more generalized stagewise superstructure to include certain configurations that both the stagewise superstructure and hyperstructure neglect. However, multiple utilities were not considered in their work.

In this work, we propose a simultaneous model based on the generalized stagewise superstructure to address HENS with multiple utilities. We let utility streams enter each stage and exit from each stage. Thus, utilities can be used to adjust the temperature of process streams in each stage. Specially, we focus on including the certain alternatives in the HEN configurations that the previous stagewise superstructure and hyperstructure neglects. We present examples in which the series matches, cross flows, and cyclic matching not accounted for the optimal placement of multiple utilities by the existing literature eliminate superior HENs. We then solve two examples to demonstrate the significant advantage of our approach and compare the results with those from the literature.

2. Superstructure and Formulation

Consider the standard HENS problem studied in the literature with usual assumptions. Our formulation is established based on a multistage superstructure with cross flows shown in Figure 1. It comprises K stages (k = 1, 2, ..., K) with |I| · |J| potential 2-stream exchangers in each stage. Each stream has one splitter at the HEN inlet and one mixer at the HEN outlet in each stage. We call these stage inlet splitters and stage outlet mixers. In addition to these, each HE has one inlet mixer and one outlet splitter for the hot-side, and likewise for the cold-side. We call these exchanger inlet mixers and exchanger outlet splitters. As it enters stage k through its own stage inlet splitter, a hot (cold) stream i (j) splits into |J| + 1 (|I| + 1) substreams. One of these substreams bypasses all the
exchangers in this stage and goes directly to the stage outlet mixer for the stream. Each of the remaining substreams enters the inlet mixer of one of HEs. Then, the two streams exiting HEs enter their respective exchanger outlet splitters. After each HE, the hot (cold) stream $i$ ($j$) splits into $f$ ($l$) substreams. Of these, one of the substreams enters their respective stage outlet mixers. The remaining substreams enter the appropriate exchanger inlet mixers of other HEs. We call these substreams cross flows. The stage outlet mixer for each hot (cold) stream receives $f + 1$ ($l + 1$) substreams, one via bypass and one each from the $f$ ($l$) HEs, to form the parent stream to the next stage. Furthermore, we let $K \geq 2$ allow possible cyclic matches between a hot and a cold process stream. Theoretically, an infinite number of infinitely small exchangers can be cyclically matched to recover energy. However, a much smaller number of cycles may be sufficient since the TAC would increase obviously with numerous HEs. Then let hot (cold) utility stream $i$ ($j$) enter stage $k$, and also exit from stage $k$. Thus, hot (cold) utilities can be used to adjust the temperature of the cold (hot) process streams in each stage.

The utility streams are different from the process streams in that their flows are unknown variables. First, using a utility stream and then letting a part of it bypass the exchangers makes no sense. Second, we do not need the stage inlet splitter for any utility stream, because utilities are assumed unlimited. Third, we assume that all utility streams will attain their final temperatures in each exchanger, so we will not need to mix them in the stage outlet mixer to attain the desired outlet temperature. This will minimize the utility flow in each exchanger. Lastly, since all utility streams have their desired outlet temperatures at the exchanger outlets, they cannot be reused, and no cross flows are needed for them. These simplifications allow us to assume that every utility stream entering an exchanger is a unique utility stream with some minimum unknown flow necessary for that exchanger.

Let $TIN_s$ ($TOUT_s$) be the temperature at which a stream $s$ enters (must exit) the HEN. Furthermore, we define $\alpha_{ijk}$ ($\gamma_{ijk}$) as the temperature of the hot (cold) stream entering $HE_{ijk}$, and $\beta_{ijk}$ ($\theta_{ijk}$) as the temperature of the hot (cold) stream leaving $HE_{ijk}$. We assume that a utility stream $s$, if it enters an exchanger, must enter (exist) at $TIN_s$ ($TOUT_s$). This is to make full use of each utility. Therefore, we have the following assignments for utility streams $i$ and $j$.

$$\alpha_{ijk} = TIN_i$$ (1)
$$\beta_{ijk} = TOUT_i$$ (2)
$$\gamma_{ijk} = TIN_j$$ (3)
$$\theta_{ijk} = TOUT_j$$ (4)
For realistic heat transfer, all stream exit temperatures in an exchanger must respect the temperature approach requirements.

\[
\begin{align*}
T_{IN_j} + MAT_{ijk} & \leq \beta_{ijk} \leq T_{IN_i} & \text{if } T_{IN_i} \geq T_{IN_j} + MAT_{ijk} \\
T_{OUT_i} & \leq \beta_{ijk} \leq T_{IN_i} & \text{otherwise} \\
T_{IN_j} & \leq \theta_{ijk} \leq T_{IN_i} - MAT_{ijk} & \text{if } T_{IN_i} \geq T_{IN_j} + MAT_{ijk} \\
T_{IN_j} & \leq \theta_{ijk} \leq T_{OUT_j} & \text{otherwise}
\end{align*}
\]

The bounds given in (5) and (6) allow that a hot (cold) process stream may be cooled (heated) beyond its desired outlet temperature at an exchanger. Huang et al. (2012) demonstrated with examples that this may result in better HENs.

Finally, our HENS formulation uses the objective of minimum TAC along with the appropriate energy and mass balances, and logical constraints.

3. Examples

We now solve two examples from the published literature to demonstrate the effectiveness of our approach. We use BARON/GAMS and limit its solution time since its convergence is slow. In order to compare our approach with those existing in the literature, we assume LMTD correction factors equal to one.

3.1. Example 1

Shenoy et al. (1998) used this example to illustrate the impact of allowing multiple utilities. It was also solved by Isafiade and Fraser (2008) and Ponce-Ortega et al. (2010). The problem has two hot (H1-H2) and one cold (C1) process streams, with high, medium and low pressure steam available as hot utilities, and cooling water as a cold utility. Figure 2 was obtained with the application of M. The HEN requires three exchangers, three heaters and two coolers. It has an investment cost of $44,057 and an annualized utility cost $52,879 to yield a TAC of $96,937. This TAC is about 0.15% lower than the best TAC of $97,079 reported by Ponce-Ortega et al. (2010) in Table 1. Their network uses only two heaters with high pressure steam and low pressure steam, but has higher utility cost. Note that at least six stages are needed for the stagewise superstructure to derive this HEN in Figure 2 for this example. Thus, the model complexity will undoubtedly increase, and the constraints may correspond to a large set, and hence the possibility of local optimum likely arises due to a large set of constraints. However, we allows cross flows in our superstructure, which is able to derive this HEN using only two stages. Thus, this example that the proposed model can give better results for HENS involving multiple utilities selection than those reported in literature.

![Figure 2. Final HEN for Example 1](image-url)
3.2. Example 2

This example was also taken from the works by Shenoy et al. (1998), Isafiade and Fraser (2008), and Ponce-Ortega et al. (2010). The problem consists of two hot process stream (H1-H2) and three cold process stream (C1-C3), with three hot utilities (high, medium, and low pressure steam) and two cold utilities (cooling water and cooling air). The optimal network (Figure 3) features a TAC of $1,115,705 for eight matches, which is 0.5% cheaper respect to the best solution reported by and Ponce-Ortega et al. (2010) in Table 2. Their HEN also uses eight HEs, but consumes more utilities. As a result, our network has less utility cost ($571,535 vs. $580,716), although it requires slightly higher heat exchanger area (4982.4 m$^2$ vs. 4926.7 m$^2$). Note that that there is a cooler at H1’s substream, similar scenario would be eliminated by the widely used assumption that utilities are used only adjust the final stream temperatures for the sake of simplicity. Thus, an HENS model with this assumption cannot admit the HEN in Figure 3 and may give suboptimal solutions for this example. As reported by Huang et al. (2012), it is worth mentioning that H1’s substream from $HE_{121}$ is at 84.9°C. which is lower than its parent stream’s final temperature of 85°C.

![Figure 3. Final HEN for Example 2](image)

Table 1. Results Comparison for Examples

<table>
<thead>
<tr>
<th></th>
<th>Example 1</th>
<th>Example 2</th>
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<tbody>
<tr>
<td></td>
<td>This work</td>
<td>Best in literature</td>
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<tr>
<td><strong>Solution method</strong></td>
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<tr>
<td><strong>HPS duty (kW)</strong></td>
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<td>238.7</td>
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<tr>
<td><strong>MPS duty (kW)</strong></td>
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<td><strong>CA duty (kW)</strong></td>
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<td>-</td>
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<tr>
<td><strong>No. of HEs</strong></td>
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<td>7</td>
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<tr>
<td><strong>TAC ($)</strong></td>
<td>96,937</td>
<td>97,079</td>
</tr>
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</table>

4. Conclusion

We presented a model based on a multistage superstructures with cross flows to address multiple utilities in HENS. Our model use to adjust the temperature of process streams in each stage. We also include certain configurations that both the stagewise superstructure and existing hyperstructure neglect. By solving two literature examples,
we demonstrate that our model admits better solutions that existing HENS formulations cannot admit.

References